$$\frac{1+\sin\theta}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}=2\tan\theta$$

$$\downarrow$$

$$=\frac{(1+\sin\theta)^2-\cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$=\frac{1+2\sin\theta+\sin^2\theta-\cos^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$=\frac{2\sin\theta+2\sin^2\theta}{\cos\theta\,(1+\sin\theta)}$$

$$=\frac{2\sin\theta\,\cancel{(1+\sin\theta)}}{\cos\theta\,\cancel{(1+\sin\theta)}}=2\tan\theta$$

$$\cot^2 y + \tan^2 y = \csc^2 y \sec^2 y - 2$$

$$\downarrow$$

$$= (1+\cot^2 y)(1+\tan^2 y) - 2$$

$$= 1 + \tan^2 y + \cot^2 y + \cot^2 y \tan^2 y - 2$$

$$= \cancel{1} + \tan^2 y + \cot^2 y + \cancel{\frac{1}{\tan^2 y} \tan^2 y - 2}$$

$$= \tan^2 y + \cot^2 y$$

Prove $(\sec x \csc x - \tan x)(\sec x \csc x - \cot x) = 1$.

$$\left(\frac{1}{\cos x}\,\frac{1}{\sin x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\,\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \left(\frac{1 - \sin^2 x}{\cos x \sin x}\right)\left(\frac{1 - \cos^2 x}{\cos x \sin x}\right)$$

$$= \frac{\cancel{\cos^2 x}}{\cancel{\cos x}\,\cancel{\sin x}}\;\frac{\cancel{\sin^2 x}}{\cancel{\cos x}\,\cancel{\sin x}}$$

$$= 1$$

$$\frac{\csc\alpha - \sec\alpha}{\sin\alpha - \cos\alpha}$$

$$= \frac{\dfrac{1}{\sin\alpha} - \dfrac{1}{\cos\alpha}}{\sin\alpha - \cos\alpha} \cdot \frac{\sin\alpha\cos\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{\overset{-1}{\cancel{\cos\alpha - \sin\alpha}}}{(\cancel{\sin\alpha - \cos\alpha})\sin\alpha\cos\alpha}$$

$$= -\csc\alpha\sec\alpha$$

$$\frac{1-\sec^2 t}{1-\cos^2 t}$$

$$= \frac{-\tan^2 t}{\sin^2 t}$$

$$= \frac{-\cancel{\sin^2 t}}{\cos^2 t} \cdot \frac{1}{\cancel{\sin^2 t}}$$

$$= -\sec^2 t$$